Short and Long Range Population Dynamics of the Monarch Butterfly

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Abstract

The monarch butterfly annually migrates from central Mexico to southern Canada. Its population has been reduced during recent decades due to human interaction with their habitat. We examine the effect of herbicide usage on the monarch butterfly's population by creating a system of non-linear ordinary differential equations that describes the interaction between the monarch's population and its environment at various stages of migration: spring migration, summer loitering, and fall migration. The model has various stages that are used to describe the dynamics of the monarch butterfly population over multiple generations. In stage 1, we propose a system of coupled ordinary different equations that models the populations of the monarch butterflies and larvae during spring migration. In stage 2, we propose a predator-prey model with age structure, larva and adult, and three classes (larvae, adult, milkweed) to model the population dynamics at the summer breeding site. In stages 3 and 4, we propose an exponential decay function to model the monarch butterfly's fall migration to central Mexico and their time at the overwintering site. The model is used to analyze the long-term behavior of the multi-stage system through numerical analysis, given data available in the research literature. As a result we observe that the actual use and an increase of herbicides has a devastating effect on the reproduction and survival of the monarch butterfly over a long period of time.

1. Background

There are several migratory populations of monarch as well, monarch butterflies east and west of the Rocky Mountain range. We focus primarily on the populations east of the Rocky Mountain range, because they have the largest population and the longest migration route. The butterflies go through four stages during one life cycle: the egg, the larvae, the pupa, and the adult butterfly.

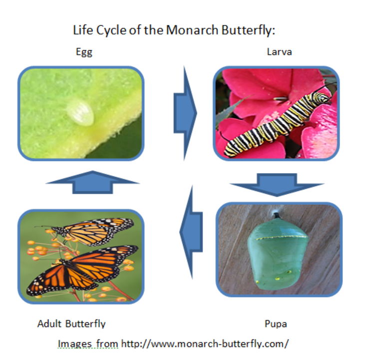


Figure 1: *Life Cycle*

1 Egg:
- If cold-hatches $\sim$ 10 days
- If hot-hatches $\sim$ 5 days

2 Larvae:
- If cold-mature slower $\sim$ 2 weeks
- If hot-mature faster $\sim <$ 2 weeks

3 Pupa: 1 week

4 Butterfly: 2-6 weeks

At the end of the life cycle, the butterfly starts its migration cycle from the overwintering site in Mexico toward southern Canada where the population will reside from June through August.

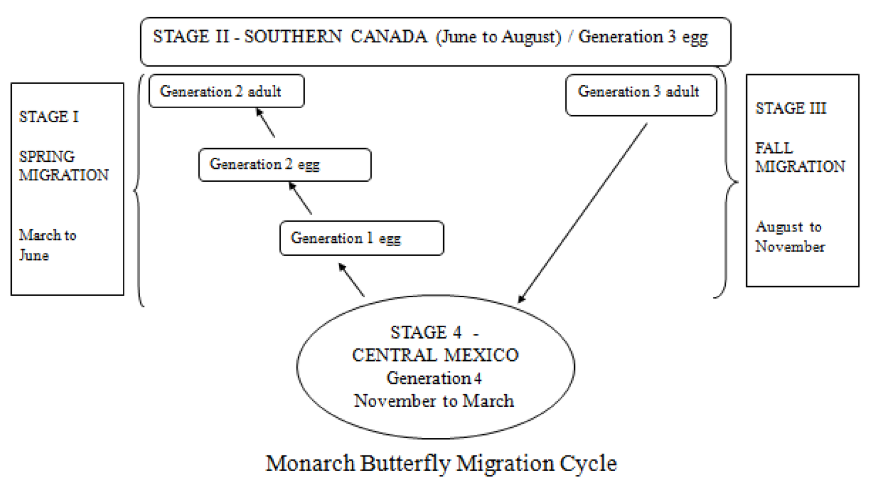


Figure 2: *Migration Cycle*

An understanding of the monarch butterfly life-cycle requires knowledge of the larvae primary food source: the milkweed. The young leaves are the prefered site for the monarch female to lay her eggs since the larvae can only eat the young leaves at their early phase.

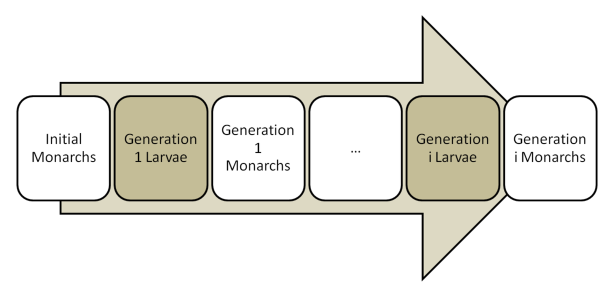
2. METHODOLOGY

The model splits the migration cycle into four stages. The first stage focuses on the monarch butterfly migration from central Mexico to southern Canada. This stage incorporates many generations since the butterfly reproduce continuously along the way.

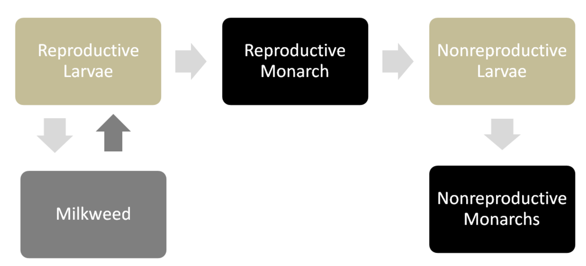
$$\frac{dM_0}{dt} = -\mu_0 M_0 \tag{1}$$
$$\frac{dL_1}{dt} = \alpha_1 M_0 A_0 - (\gamma + \mu_1) L_1 \tag{2}$$
$$\frac{dM_1}{dt} = \gamma L_1 - \mu_2 M_1 \tag{3}$$
$$\cdots$$
$$\frac{dL_i}{dt} = \alpha_1 M_{i-1} A_0 - (\gamma + \mu_1) L_i \tag{4}$$
$$\frac{dM_i}{dt} = \gamma L_i - \mu_2 M_i \tag{5}$$

The second stage describes the butterfly in Canada and considers the effect of herbicide on the milkweed. For L_s the larvae which transform to overwintering adults, and M_s the overwintering adults, we have the initial conditions $L_s = 0$, $M_s = 0$, and $0 < A < 2$.

$$\frac{dL_{in}}{dt} = -(\gamma + \mu_1) L_{in} \tag{6}$$
$$\frac{dM_{in}}{dt} = -\mu_2 M_{in} + \gamma L_{in} \tag{7}$$
$$\frac{dL_s}{dt} = \alpha_2 A M_{in} - (\gamma + \mu_1) L_s \tag{8}$$
$$\frac{dM_s}{dt} = \gamma L_s - \mu_3 M_s \tag{9}$$
$$\frac{dA}{dt} = aA\left(1 - \frac{A}{K}\right) - A(\sigma + \beta L_s) \tag{10}$$



(a) Pictorial representation of *stage 1*.



(b) Pictorial representation of *stage 2*.

Figure 3:

The third stage illustrates the monarchs journey back to central Mexico. Because a lot of butterflies will not reach central Mexico due to natural catastrophes, weather, and other environmental factors, we have an exponential decay function of the form:

$$\frac{dM_{fin}}{dt} = -\mu_{fin} M_{fin}$$

The last stage model the butterfly at its dormant phase. where they will only start reproducing. Since the monarch are in state of reproductive dipause and big portion of them will die, we have a similar situation as in stage 3 where the population that arrives in Mexico ($M_w(0)$) represents the initial condition for the following equation:

$$\frac{dM_w}{dt} = -\mu_w M_w$$

Then a new cycle begins starting in March with a similar pattern.

3. Results

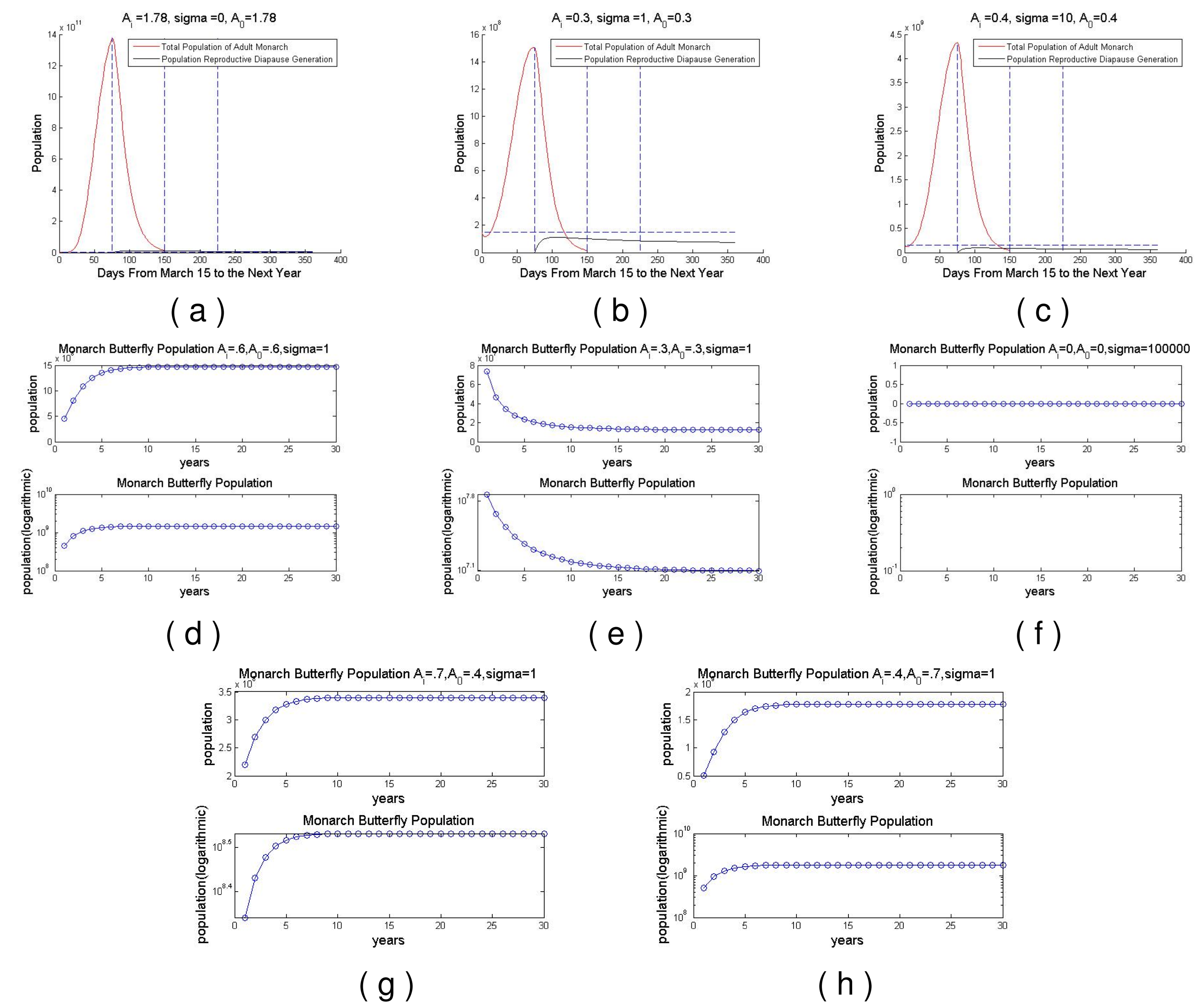


Figure 4: *Short and long range dynamics of the monarch butterfly for with various values of* A_0 *and* σ*.*

In figures (a), (b), and (c), we consider a time interval of one year. For a constan initial monarch population of 150 000 000 at the overwintering site, we predict the change of both the populations size and its behavior under different values of herbicide factor and intial milkweed population. We also look at the behavior of the population over 30 years shown in graph (d), (e), and (f) with different herbicide numbers.

Finally we examine the impact of the population size in the last generation in stage 1 on the persistence or extinction of the monarch butterfly over a long period of time (graph g and h).

4. Conclusion

A mathematical modeling of the monarch butterfly requires extensive background knowledge of its life-cycle. Through our simulation we examine the impact of the herbicide on the monarch population. As the herbicide factor grows, the monarch population decreases. This research provides an approximation of the herbicide usage that can lead to extinction of the monarch butterfly population. It will be favorable to create a monarch way station throughout different city from the south to the north of United States so we can prevent the extinction of the butterfly and conserve the beauty of our nature.

5. Acknowledgments

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