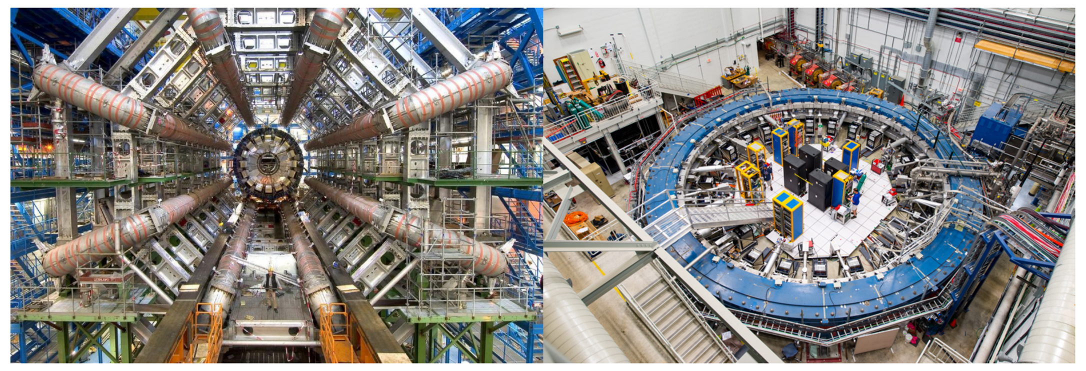
Digital Quantum Simulation of Gauge Theories

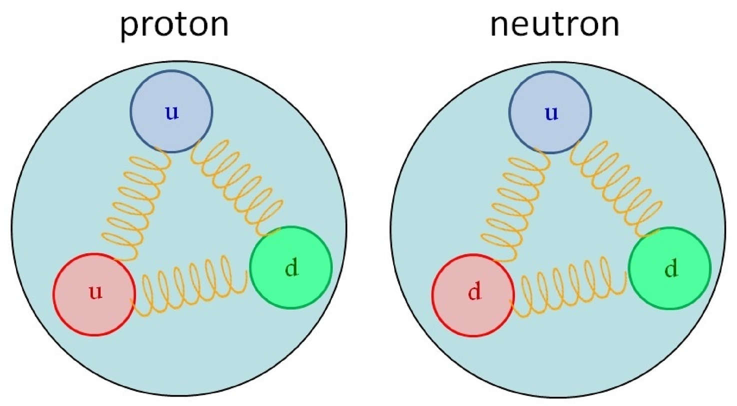
Yao Ji, Henry Lamm, and Shuchen Zhu
arXiv:2005.14221/PRD arXiv:2203.02330
Technical University of Munich TUM
Fermilab
Georgetown University

Introduction

- The field of HEP relies on lattice QCD for precision calculations.

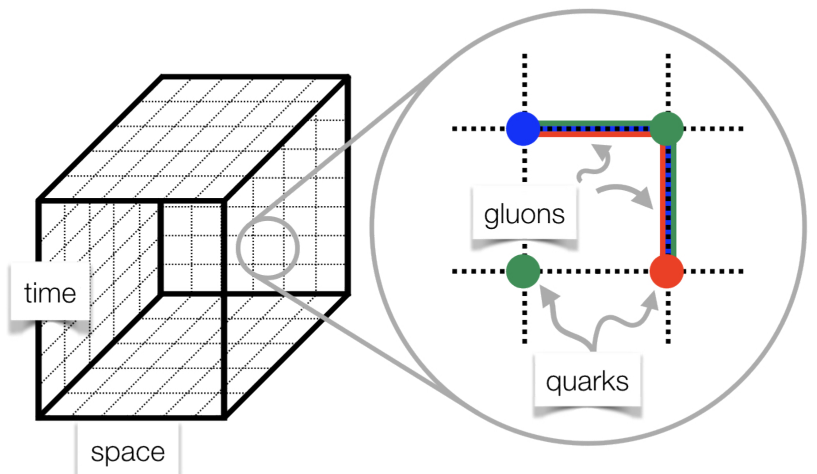


- Important result are extracted from lattice QCD. Unfortunately, there is restriction of problems that can be solved classically because $BPP \subsetneq BQP$ [1].



- Quantum computers can simulate the time evolution, but the exact resource required is unknown. A current resource estimation [2] suggests a loose upper bound $\mathcal{O}(10^{49})$ of T gates.
- We are looking into *digitization schemes* for Hamiltonians such that quantum resources can be saved immensely.

Classical Lattice QCD



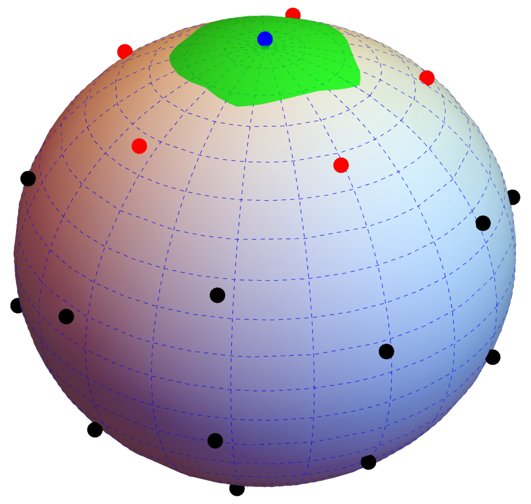
- Discretize the 4-dimensional space time into 4-dimensional Euclidean lattice.
- Classical lattice allows nonperturbative study of truncated Hamiltonian before quantum computers become available.

Methods

The high-level idea is to approximate the partition function over a continuous gauge group G by an effective one over the discrete subgroup H of G. We showed two schemes for digitizing the $SU(N)$ gauge group of an effective action to any arbitrary order.

Groups Space Decimation. In this way the partition function integrating over G can be written, without approximation, as a summation over H and integration over fluctuations:

$$Z = \int_G DU\, e^{-S[U]} = \sum_{u\in H} \int_\Omega D\epsilon\, e^{-S[u,\epsilon]}\,.$$



Character Expansion. The general idea is to approximate the partition function, which can be written as a series expansion over infinitely many $SU(3)$ characters, by an effective one over a finite number of $S1080$ group characters,

$$e^{-S[U]} = \sum_{(\lambda,\mu)} \beta_{(\lambda,\mu)} \chi_{(\lambda,\mu)}(U), \quad U \in SU(3).$$

For the defined Wilson action, each expansion coefficient $\beta_{(\lambda,\mu)}$ are calculable as an infinite sum in terms of Bessel functions. Thus we could obtain an effective partition function over $S1080$ by matching the following ansatz

$$e^{-\widetilde{S}[u]} = \sum_r \beta'_r \chi'_r(u), \quad u \in S1080$$

to the previous equation hence determining the effective action $\widetilde{S}$ to the target accuracy.

Results

The following plots show the average energy per plaquette, $\langle E_0 \rangle = 1 - \Re\langle \mathrm{Tr}\, U_p\rangle/3$, vs $\tilde{\beta}_1$ on 4^4 lattice for $S1080$ action with corrections. The black line is the $SU(3)$ result.

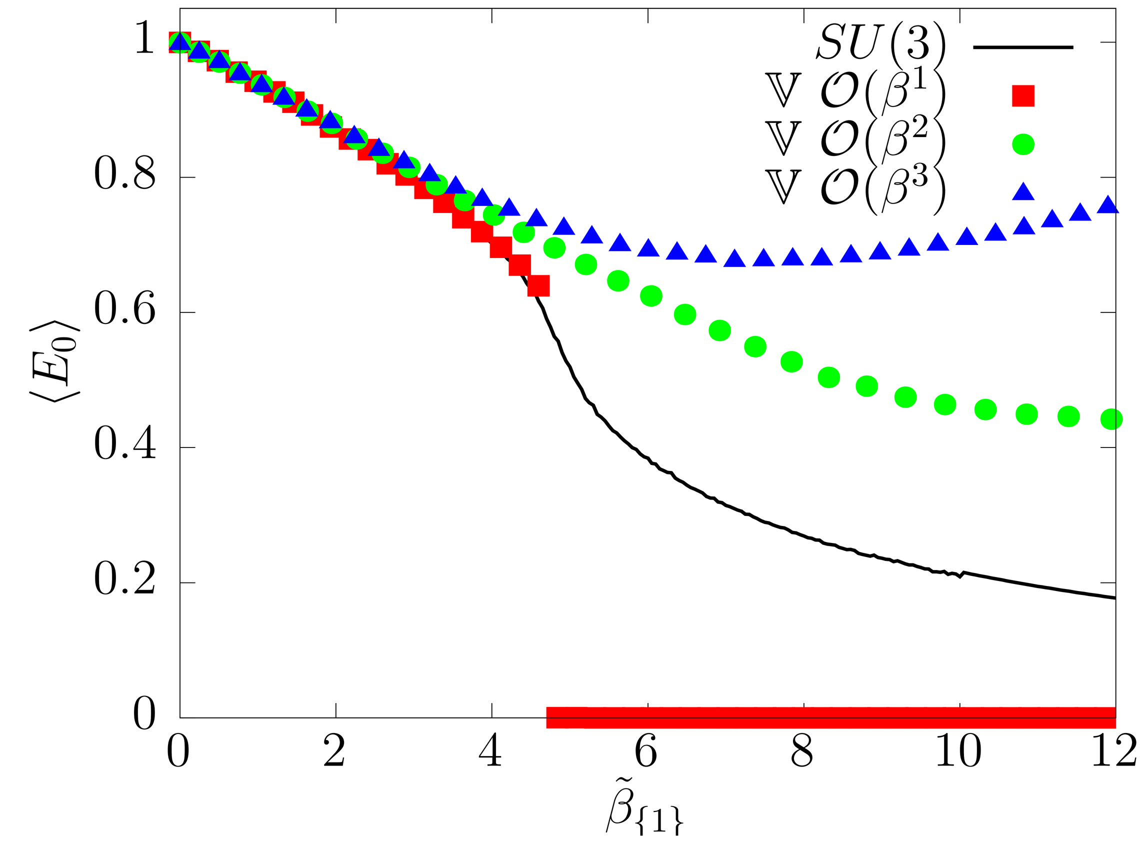


Figure 1: By group space decimation.

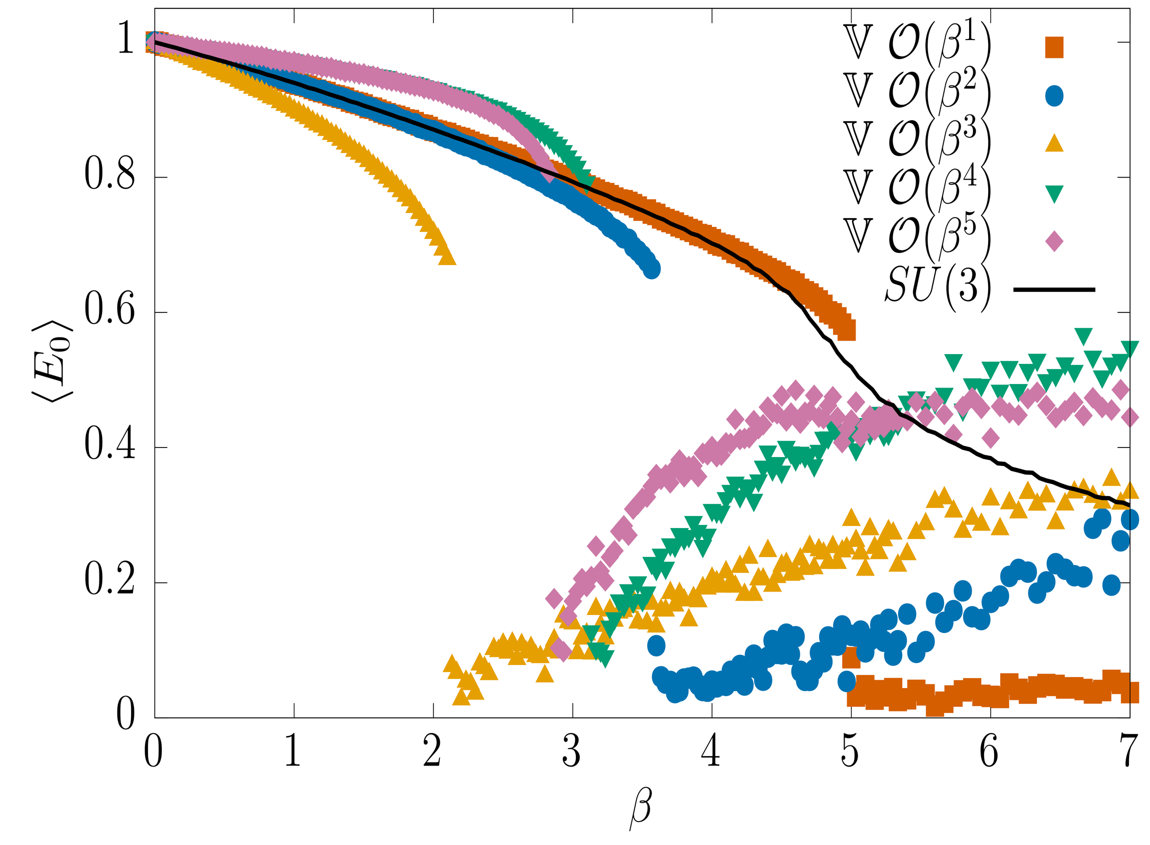
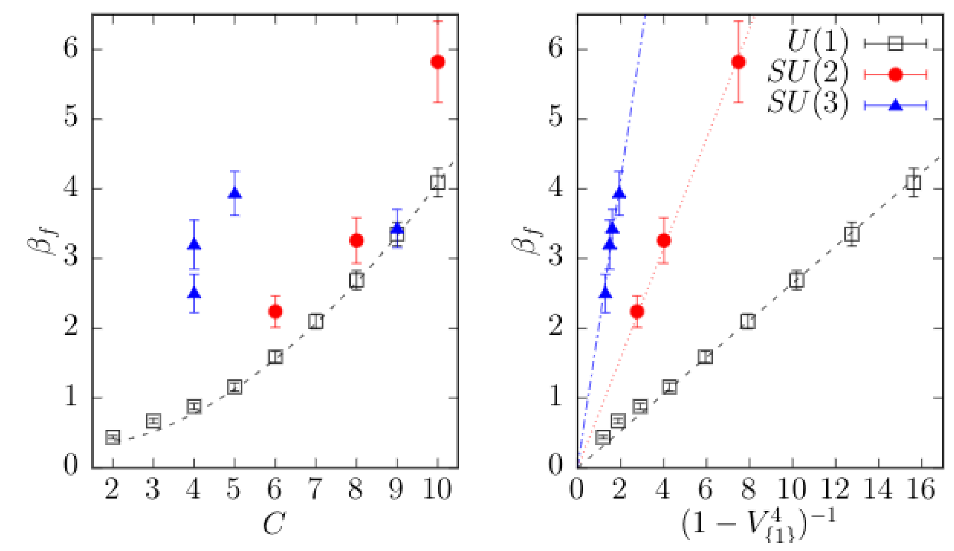


Figure 2: By character expansion.

Freezing Point Prediction

$(1 - V^4_{\{1\}})^{-1}$ is exceptionally good at predicting β_f across gauge groups.

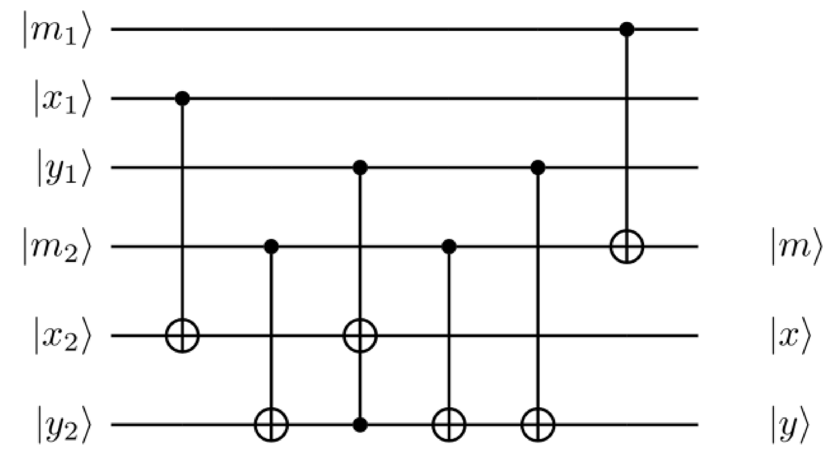


Discussion

- Both schemes faithfully produce the physical results below the freezing points.
- Both schemes can be generalized to arbitrary higher-order for required accuracy.
- The *group space decimation* has predictive power determining the freezing point.
- The equivalent Hamiltonian from these schemes requires fewer qubits for each trottfer step simulation.

Ongoing Research

Another important step in studying the feasibility of these procedures is to explicitly construct the quantum registers and primitive gates that operate over smaller discrete groups [3].



$$g_1 g_2 = \omega^{p+p'+q'r+q'rs-q'rt-q'r't-rr't-q'rst-rr'st} \times C^{q+q'+sq'-tq'-stq'-tr'-str'} \times E^{r+tq'+stq'+r'+sr'-tr'-str'} V^{2(s+s')+t+t'}$$

References

[1] Stephen P Jordan, Hari Krovi, Keith SM Lee, and John Preskill. *Quantum*, 2:44, 2018.
[2] Angus Kan and Yunseong Nam. *arXiv preprint arXiv:2107.12769*, 2021.
[3] Andrei Alexandru, Paulo F Bedaque, Siddhartha Harmalkar, Henry Lamm, Scott Lawrence, Neill C Warrington, NuQS Collaboration, et al. *Physical Review D*, 100(11):114501, 2019.