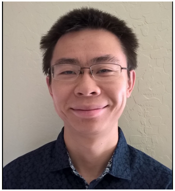
Gluon Field Digitization via Group Space Decimation for Quantum Computers

Shuchen Zhu
Georgetown University

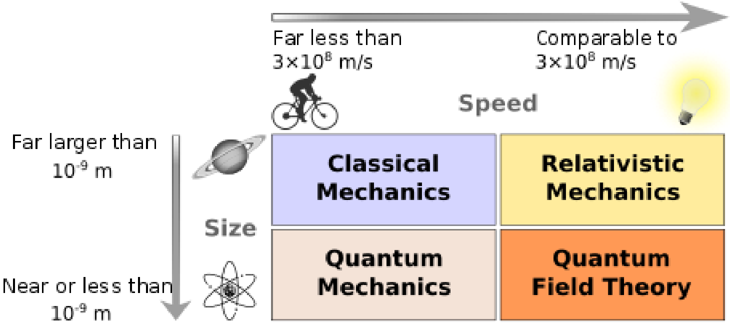


Yao Ji
University of Siegen

Henry Lamm
Fermi National Lab

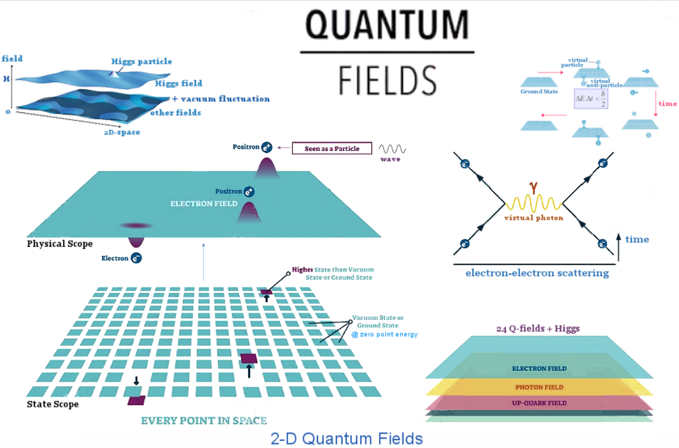
arXiv:2005.14221/PRD

Theories for different scales



Quantum field theory (QFT) is a theoretical framework that combines classical field theory, special relativity and quantum mechanics.

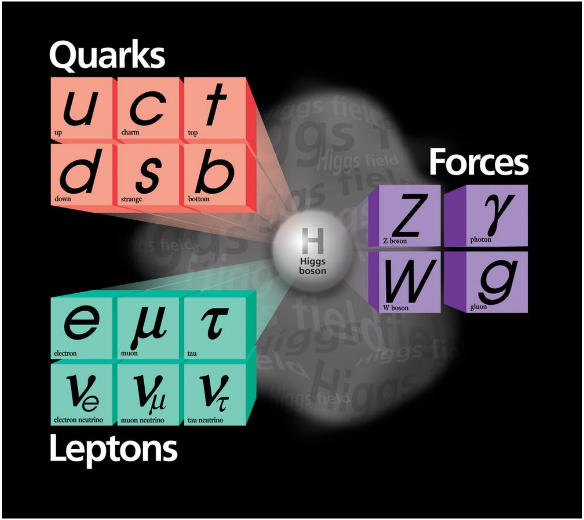
View of quantum field theory

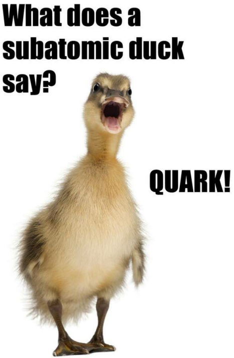


Particles are excitation of the underlying fields, and interactions are described by Feynman Diagrams involving their corresponding quantum fields.

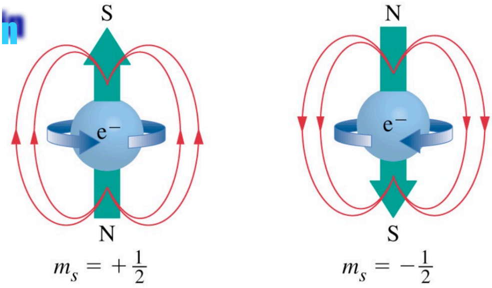
What does QFT give us?

The Standard Model





An accurate theory



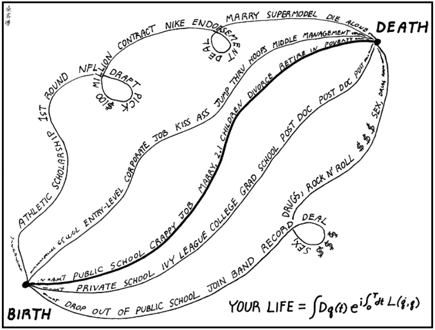
The best measured value of anomalous magnetic moment of the electron is

$$g/2 = 1.00115965218085(76).$$

- QFT for electrodynamics predicted all these decimal values!
- The QFT for electrodynamics is the most precisely tested theory in the history of science.

Hard to solve analytically

The continuous path integral (for strong nuclei interaction) is very hard to evaluate, analytically.

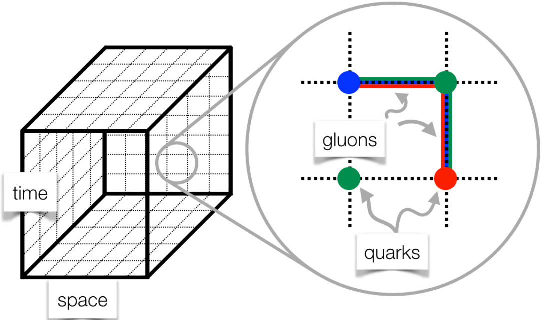


The Path Integral Formulation of Your Life

Path integral is functional integral over an infinity of quantum-mechanically possible trajectories to compute a quantum amplitude.

Solve it numerically

Discretize the 4-dimensional space time into 4-dimensional Euclidean lattice.



Extrapolate the physical values in the end, by setting the lattice spacing $a \to 0$.
This is Lattice Quantum Chromodynamics (QCD) on classical computers.

Current limitations

- Cannot simulate the real-time dynamics of a quark-gluon system.
- It is computationally intensive, needs huge memory access bandwidth.
- Gives reliable prediction only for certain models.

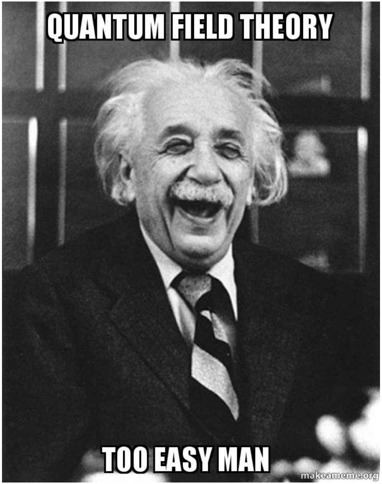
Current limitations

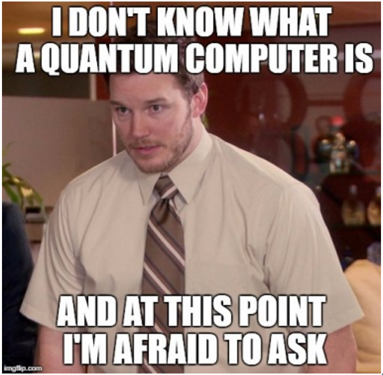
- Cannot simulate the real-time dynamics of a quark-gluon system.
- It is computationally intensive, needs huge memory access bandwidth.
- Gives reliable prediction only for certain models.

It takes 50 years of development:
Formulate the problem (1970s)
Reaching the continuum (1980s)
Reducing lattice artifacts (1990s)
Dynamical Fermions (2000s)
Form factors, QED (2010s)
Nuclei (2020s)

A new 50-year adventure?

Quantum field theory + Quantum Computer





Just need a few steps...

There are four broad steps for quantum simulation of QFT:

- Digitization: how gluon fields are represented on quantum registers?
- Initialization: how to set quantum registers as actual physical state?
- Propagation: how gates evolve physical state? (work in progress)
- Evaluation: how to compute the observables?

Just need a few steps...

There are four broad steps for quantum simulation of QFT:

- Digitization: how gluon fields are represented on quantum registers?
- Initialization: how to set quantum registers as actual physical state?
- Propagation: how gates evolve physical state? (work in progress)
- Evaluation: how to compute the observables?

We focus on finding an efficient digitization scheme.

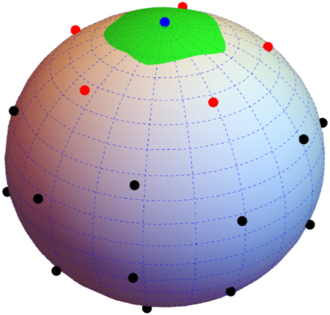
- What are the quantum resources required?
- What is the rate of approach to the physical point?
- Can the scheme be simulated classically?

Digitizing gluon fields

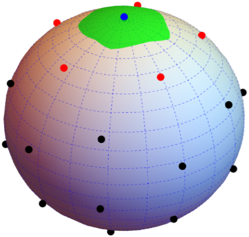
By decimating group structure for the gluon field:

$$SU(3) \to S1080,$$

$S1080$ is the largest discrete subgroup of $SU(3)$.



Group space decimation



The path integral becomes

$$\int_{SU(3)} DU\, e^{-S[U]} = \sum_{u \in S1080} e^{-S[u]},$$

where showing that $S[U]$ on the LHS can be replaced by $S[u]$ is a key technical contribution.

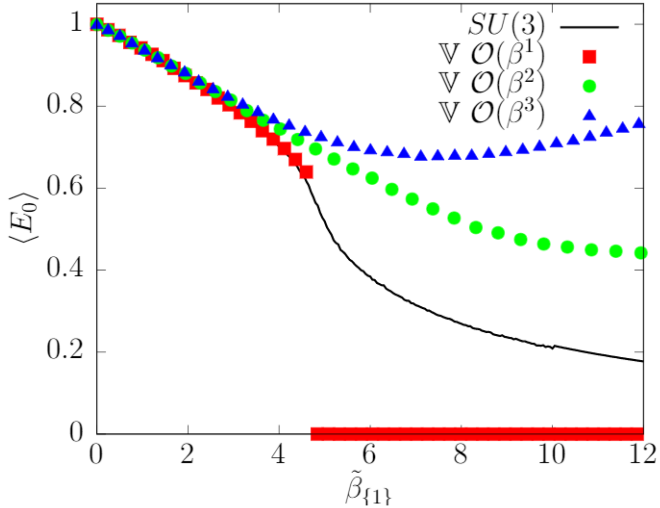
The decimated action

The effective decimated action $S[u]$ in the path integral can be approximated (systematically) to any order. The third order approximation reads

$$S[u] = \sum_p -\left(\beta_{\{1\}} + \beta_{\{1,1\}}\right)\frac{1}{3}\,\mathrm{Re}\,\chi_{\{1\}}(u) - \left(\beta_{\{0\}} + \beta_{\{1,1,1\}}\right)$$
$$-\left(\beta_{\{2\}} + \beta_{\{1,1,-1\}}\right)\frac{1}{6}\,\mathrm{Re}\,\chi_{\{2\}}(u)$$
$$-\left(\beta_{\{1,-1\}} + \beta_{\{2,1\}}\right)\frac{1}{8}\chi_{\{1,-1\}}(u)$$
$$-\frac{\beta_{\{3\}}}{10}\,\mathrm{Re}\,\chi_{\{3\}}(u) - \frac{\beta_{\{2,-1\}}}{15}\,\mathrm{Re}\,\chi_{\{2,-1\}}(u)\,,$$

where χ_r's are the character functions for $u \in S1080$.

How does the scheme do?

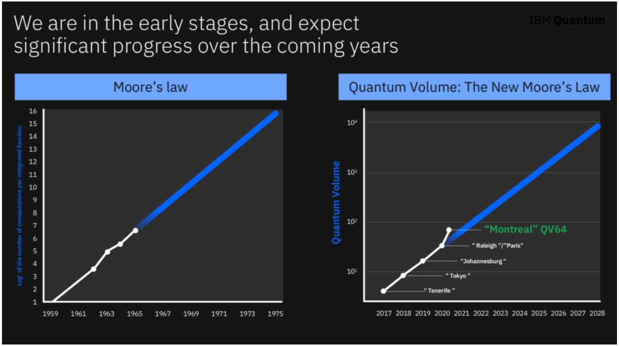


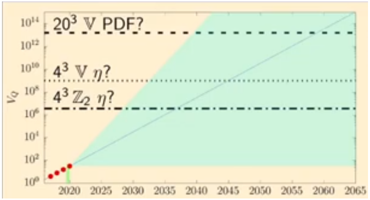
Why this scheme

We used group space decimation as the scheme for efficient digitization of gluon field.

- What are the quantum resources required?
  A: It saves 100 times as many quantum bits for each gluon link.
- What is the rate of approach to the physical point?
  A: The first order approximation gives sufficient accuracy at low energy scale.
- Can the scheme be simulated classically?
  A: Yes, and that was how we did the experiment.

A bright future?





Future work

- Try to synthesize quantum circuits to evolve quantum state.
- Find other approximation schemes.