EXAM 1

Math 212, 2019 Fall, Clark Bray.

Name:____ Section:______ Student ID:____________

GENERAL RULES

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

No notes, no books, no calculators.

All answers must be reasonably simplified.

All of the policies and guidelines on the class webpages are in effect on this exam.

WRITING RULES

Do not write anything near the staple – this will be cut off.

Use black pen only. You may use a pencil for initial sketches of diagrams, but the final sketch must be drawn over in black pen and you must wipe all erasure residue from the paper.

Work for a given question can be done ONLY on the front or back of the page the question is written on. Room for scratch work is available on the back of this cover page, and on the two blank pages at the end of this packet; scratch work will NOT be graded.

DUKE COMMUNITY STANDARD STATEMENT

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ________________________

(Scratch space. Nothing on this page will be graded!)

1. *(20 pts)*

   (a) Find the area of the parallelogram defined by $\vec{v} = (1, 2, 0)$ and $\vec{w} = (3, 0, 4)$.

   (b) Compute the angle between $\vec{v}$ and $\vec{w}$.

   (c) The plane $-8x + 4y + 6z = 5$ passes through a point whose position vector is $\vec{x}_0$. Is the list $\vec{x}_0$, $\vec{v}$, $\vec{w}$ in right hand order or left hand order, or neither? *(Hint: Consider how you can use some of the work from part (a). And be sure to explain your reasoning!)*

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2. *(20 pts)* A particle has initial position $\vec{x}_0 = (1, 2, 3)$ and velocity given by $\vec{v}(t) = (6t^2, 3e^t, 4 \sin t)$.

   (a) Find an expression for the acceleration of the particle as a function of time.

   (b) Find an expression for the position of the particle as a function of time.

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3. *(20 pts)* The surface S has equation $x^2 + (2y-4)^2 - z^2 = 1$. Describe S by identifying

(a) a rotationally symmetric surface R (and explain how you know what R looks like); and

(b) a sequence of geometric transformations that turn R into S.

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4. *(20 pts)*

(a) Find an equation for the graph of the function $f : \mathbb{R}^3 \to \mathbb{R}^1$ defined by $f(v, w, x) = v^2w - wx^3$.

(b) Find a function $h : \mathbb{R}^n \to \mathbb{R}^m$ for which one of the level sets is the sphere with equation $(x+1)^2 + y^2 + z^2 = 9$, and identify the corresponding values of n and m.

(c) Identify a level set of the function g (defined by $g(x, y) = (y - x^2)^2$) that is also the graph of another function $p : \mathbb{R}^a \to \mathbb{R}^b$, and identify that other function p and the values a and b.

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5. *(20 pts)*

(a) We are given that $z = x^2 - y^2$, $x = 3r + 2s$, and $y = r - 5s$. Find an expression for $\dfrac{\partial z}{\partial s}$ in terms of the partials of z with respect to x and y. *Reminder – do NOT plug in to find z explicitly as a function of r and s!*

(b) Suppose now that $r = 2\cos t$ and $s = 3\sin t$. Compute the value of $\dfrac{dz}{dt}$ when $t = 0$.

(Hint: Evaluate $\dfrac{dr}{dt}(0)$ first.)

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