

# Math 212

## Additional Homework Problems

1. Let  $C$  be the unit circle in the  $xy$ -plane, oriented counterclockwise as seen from above. The divergence of the vector field  $\vec{F} = (z, x, y)$  is zero, and as a result the flux through every surface with boundary  $C$  should be the same. Confirm that this is the case with the upper half of the unit sphere, the lower half of the unit sphere, and the unit disk in the  $xy$ -plane.
2. Compute the flux of the vector field  $\vec{F} = (z - x^2 + 3xy^2, 2xy - y^3, 2xye^x)$  through the surface  $S$  that is the part of the surface  $x^2 + z^4 + y = 5$  with  $y \geq 0$ , oriented in the positive  $y$  direction.
3. Compute the flux of the vector field  $\vec{F} = (x^2 + 5x - yz, 6 - 2xy, e^y - 5z)$  through the surface  $S$  that is the part of the surface  $x = y^2 + z^2 - 4$  with  $x \leq 0$ , oriented in the negative  $x$  direction.
4. Compute the flux of the vector field  $\vec{F} = (e^z - 2xy, y^2 - e^z, 2xy - y^2)$  through the surface  $S$  that is the part of the surface  $z = x^4 + e^{y^2}$  with  $x + y + z \leq 1$ , oriented upward.
5. Suppose we consider the vector field  $\vec{F}(x, y, z) = (x^2y - yz, xy - y^2, xz)$ . Is the point  $(1, 2, 1)$  a vector field source or a vector field sink? Explain your answer.
6. The flow of locusts is given by  $\vec{F}(x, y) = (5x - y, x - 2y)$  (in millions of locusts per mile per day). Square county is located over the region  $S = [0, 2] \times [2, 4]$ . If we measure the rate of change of the number of locusts in a region as the flux through the boundary curve of the vector field  $\vec{F}$ , how fast is the number of locusts in Square county changing?
7. A small spherical balloon is held in position at the point  $(1, 2, 3)$ , but is free to rotate under the influence of the air current described by the vector field  $\vec{F}(x, y, z) = (y - z^2, 3x + y + z^2, x^2 - 2y - z)$  giving wind velocity in terms of position. What is the axis around which the balloon rotates?

If the balloon were moved to the point  $(0, 0, 0)$ , would it rotate faster or slower, and by what factor?